a)
$$f(x)=\frac{1+a\cos x}{\sin x-b\cos x-3}$$

$\left(0;-\frac{1}{3}\right)$

$(\pi;0)$

$$\begin{cases}-\frac{1}{3}=\dfrac{1+a\cos 0}{\sin 0-b\cos 0-3}\\[2ex] 0=\dfrac{1+a\cos\pi}{\sin\pi-b\cos\pi-3}\end{cases}\qquad\begin{cases}-\frac{1}{3}=\dfrac{1+a}{-b-3}\\[2ex] 0=\dfrac{1-a}{b-3}\end{cases}$$

Dalla seconda per $b\neq 3\Rightarrow a-1=0\Rightarrow \underline{a=1}$

$$-\frac{1}{3}=\frac{-2}{b+3}\Rightarrow \frac{1}{3}=\frac{2}{b+3}\Rightarrow b+3=6\Rightarrow \underline{b=3}$$

c) $$f(x)=\frac{1+\cos x}{\sin x-3\cos x-3}$$

Si divide numeratore e denominatore per $1+\cos x$

$$f(x)=\frac{\dfrac{1+\cos x}{1+\cos x}}{\dfrac{\sin x}{1+\cos x}-\dfrac{3(\cos x+1)}{\cos x+1}}\qquad f(x)=\frac{1}{\dfrac{\sin x}{1+\cos x}-3}$$

$$f(x) = \frac{1}{tg\frac{x}{2} - 3}$$

Essendo $tg\frac{x}{2} = \frac{sin x}{1+\cos x}$

Se $tg\frac{x}{2} = t$ si ottiene $g(t) = \frac{1}{t-3}$

$g(t)$ è un'omografica del tipo $g(t) = \frac{\bar{a}t + \bar{b}}{\bar{c}t + \bar{d}}$

In questo caso $\bar{a} = 0$

$\bar{b} = 1$

$\bar{c} = 1$

$\bar{d} = -3$

$\begin{cases} \text{Asintoto verticale: } t=3 \\ \text{Asintoto orizzontale } g(t)=0 \end{cases}$