$$\int \frac{\log(1+x)}{x}dx = \int \frac{\sum_{n=0}^{\infty}(-1)^n \frac{x^{n+1}}{n+1}}{x}dx = \int \sum_{n=0}^{\infty}(-1)^n \frac{x^n}{n+1}dx =$$

$$= \sum_{n=0}^{\infty}(-1)^n \left[\int \frac{x^n}{n+1}dx\right] = \sum_{n=1}^{\infty}(-1)^n \frac{x^{n+1}}{(n+1)^2} + C = x - \frac{x^2}{4} + \frac{x^3}{9} \cdots + C$$

$$\int sin\sqrt{x-2}\,dx = \int \sum_{n=0}^{\infty}(-1)^n \frac{\left(\sqrt{x-2}\right)^{2n+1}}{(2n+1)!}dx = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\int \left(\sqrt{x-2}\right)^{2n+1} dx =$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\int (x-2)^{n+\frac{1}{2}}dx = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\frac{(x-2)^{n+\frac{3}{2}}}{n+\frac{3}{2}} + C =$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n \cdot 2}{(2n+3)(2n+1)!}(x-2)^{n+\frac{3}{2}} + C = 2(x-2)^{\frac{1}{2}}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+3)(2n+1)!}(x-2)^{n+1} + C =$$

$$2\sqrt{x-2}\left[\frac{x-2}{3} - \frac{(x-2)^2}{5\cdot 3!} + \frac{(x-2)^3}{7\cdot 5!}\cdots\right] + C$$

Nota:

$ln(1+x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{n+1}}{n+1} = x - \frac{x^2}{2} + \frac{x^3}{3} \ldots$

$senx = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots$ perciò

$sen\sqrt{x-2} = \sum_{n=0}^{\infty}(-1)^n \frac{(\sqrt{x-2})^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\int (x-2)^{n+\frac{1}{2}}dx = \sqrt{x-2} - \frac{\sqrt{(x-2)^3}}{3!} + \frac{\sqrt{(x-2)^5}}{5!} \ldots$