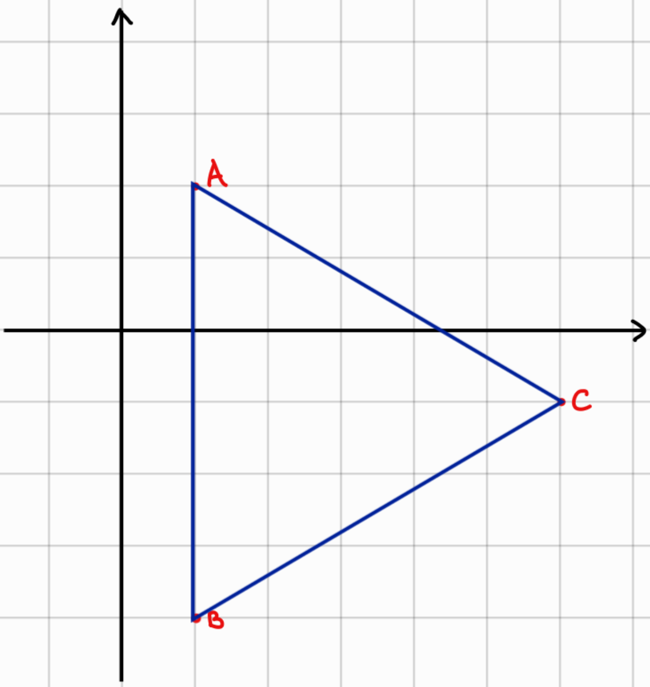
$A(1,2)$

$B(1,-4)$

$C(6,-1)$



Dato che la base è AB bisogna calcolare la lunghezza dei lati $\overline{AC}$ e $\overline{BC}$ e verificare che siano uguali

$$\overline{BC} = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2} = \sqrt{(6-1)^2 + [-1-(-4)]^2} = \sqrt{5^2 + [-1+4]^2} = \sqrt{5^2 + 3^2} = \sqrt{25+9} = \sqrt{34}$$

$$\overline{AC} = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2} = \sqrt{(6-1)^2 + (-1-2)^2} = \sqrt{5^2 + (-3)^2} = \sqrt{25+9} = \sqrt{34}$$

$\overline{AC} = \overline{BC} \Rightarrow$ Il triangolo è isoscele

$$A = \frac{b \cdot h}{2} = \frac{\overline{AB} \cdot \overline{MC}}{2}$$

$$x_M = \frac{x_A + x_B}{2} = \frac{1+1}{2} = 1 \qquad y_M = \frac{y_A + y_B}{2} = \frac{2-4}{2} = -1 \qquad M = (1,-1)$$

ovvio perché uguale a y_C

$$\overline{MC} = \sqrt{(x_C - x_M)^2 + (y_C - y_M)^2} = \sqrt{(6-1)^2 + [-1-(-1)]^2} = \sqrt{5^2 + [-1+1]^2} = \sqrt{5^2 + 0} = 5$$

$$\overline{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{(1-1)^2 + (-4-2)^2} = \sqrt{0 + (-6)^2} = \sqrt{6^2} = 6$$

$$A = \frac{\overline{AB} \cdot \overline{MC}}{2} = \frac{6 \cdot 5}{2} = 15$$

oppure l'area di qualunque triangolo si può calcolare anche così:

$$A = \frac{1}{2}\begin{vmatrix} x_B - x_A & y_B - y_A \\ x_C - x_A & y_C - y_A \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1-1 & -4-2 \\ 6-1 & -1-2 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 0 & -6 \\ 5 & -3 \end{vmatrix} = \frac{1}{2} \cdot 30 = 15$$

N punto medio di AC

$$x_N = \frac{x_A + x_C}{2} = \frac{1+6}{2} = \frac{7}{2} \qquad y_N = \frac{y_A + y_C}{2} = \frac{2-1}{2} = \frac{1}{2} \qquad N\left(\frac{7}{2}, \frac{1}{2}\right)$$

$$\overline{NB} = \sqrt{(x_B - x_N)^2 + (y_B - y_N)^2} =$$

$$= \sqrt{\left(1 - \frac{7}{2}\right)^2 + \left(-4 - \frac{1}{2}\right)^2} = \sqrt{\left(\frac{2-7}{2}\right)^2 + \left(\frac{-8-1}{2}\right)^2}$$

$$= \sqrt{\left(-\frac{5}{2}\right)^2 + \left(-\frac{9}{2}\right)^2} = \sqrt{\frac{25+81}{4}} = \frac{\sqrt{106}}{2}$$

Il triangolo APB deve essere equilatero $P(x,-1)$

$\overline{AB} = 6$

$\overline{AP} = \overline{BP} = 6$

$$\overline{AP} = \sqrt{(x_P - x_A)^2 + (y_P - y_A)^2} =$$

$$= \sqrt{(x-1)^2 + (-1-2)^2} =$$

$$= \sqrt{x^2 - 2x + 1 + 9} = \sqrt{x^2 - 2x + 10}$$

$$\sqrt{x^2 - 2x + 10} = 6$$

$$x^2 - 2x + 10 = 36$$

$$x^2 - 2x - 26 = 0 \qquad x_{1,2} = 1 \pm \sqrt{1+26} = 1 \pm \sqrt{27}$$