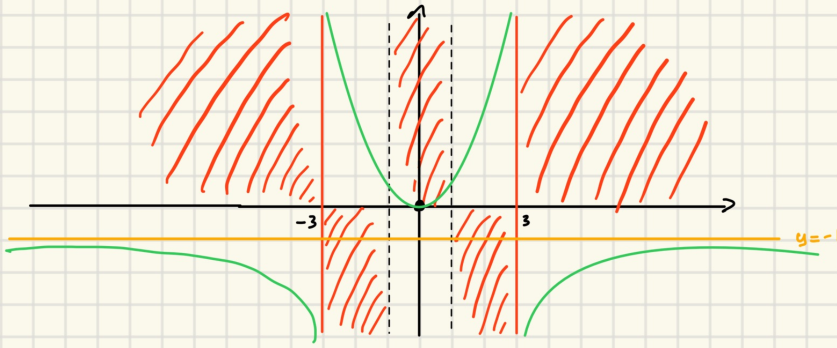
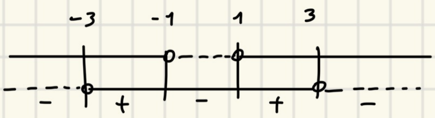
$f(x) = \frac{x^2-1}{9-x^2}$



1. Dominio: $9-x^2 \neq 0 \to x \neq \pm 3$ $\quad D_f: \mathbb{R} - \{\pm 3\}$

2. Studio del segno:

$$f(x) \geq 0 \to \frac{x^2-1}{9-x^2} \geq 0 \quad \begin{cases} x^2-1 \geq 0 \to x \leq -1 \vee x \geq 1 \\ 9-x^2 > 0 \to x^2-9 < 0 \to -3 < x < 3 \end{cases}$$



Pertanto, $f(x) > 0$ se $x \in \left]-3;-1\right] \cup \left[1;3\right[$

$f(x) < 0$ se $x \in \left]-\infty;-3\right[\cup \left]-1;1\right[\cup \left]3;+\infty\right[$

3. Asintoti verticali

$$\lim_{x \to -3^-} \frac{x^2-1}{9-x^2} = \frac{8}{0^-} = -\infty \qquad \lim_{x \to 3^-} \frac{x^2-1}{9-x^2} = \frac{8}{0^+} = +\infty$$

$$\lim_{x \to -3^+} \frac{x^2-1}{9-x^2} = \frac{8}{0^+} = +\infty \qquad \lim_{x \to 3^+} \frac{x^2-1}{9-x^2} = \frac{8}{0^-} = -\infty$$

Asintoti orizzontali:

$$\lim_{x \to -\infty} \frac{x^2-1}{9-x^2} = \frac{\cancel{x^2}\left(1-\cancel{\frac{1}{x^2}}\right)}{\cancel{x^2}\left(\cancel{\frac{9}{x^2}}-1\right)} = -1 \qquad \lim_{x \to +\infty} \frac{x^2-1}{9-x^2} = -1$$

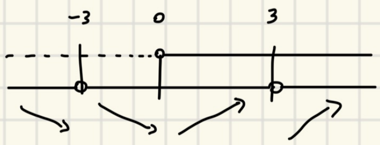
$y=-1$ Asintoto orizzontale completo.

4. Punti stazionari

$$f'(x) = \frac{2x(9-x^2)-(x^2-1)(-2x)}{(9-x^2)^2} = \frac{2x(9-\cancel{x^2}+\cancel{x^2}-1)}{(9-x^2)^2} \to \frac{16x}{(9-x^2)^2}$$

$$f'(x) = 0 \to \frac{16x}{(9-x^2)^2} = 0 \to x = 0$$

$$f'(x) > 0 \to \frac{16x}{(9-x^2)^2} > 0 \quad \begin{cases} x > 0 \\ x \neq \pm 3 \end{cases}$$



$x=0$ Punto di minimo.